Gauge Theories of Gravity: Teleparallelism and Nonlocality

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Comprehensive Exam, September 26, 2000

Overview

- Special relativity: simultaneity, length measurement, accelerated observers
- Hypothesis of Locality and its problems
- Electrodynamics and accelerated observers
- Equivalence principle and gauge theories of gravity
- Teleparallel Theories

Main topics

1. Problems with traditional treatment of accelerated observers, using the Hypothesis of Locality: length measurements

2. Alternative nonlocal approaches in electrodynamics

3. Alternative to traditional application of Principle of Equivalence: teleparallel theories

SR: Simultaneity and Lengths

Event: single location/position, single instant in time.

Position of event: coordinate label on an indefinitely extended rigid ruler.

Time of event: Reading on a clock located at position of event

Inertial observers can use synchronized clocks: They can correct for travel time of a signal. Prior knowledge needed: Distance between source of signal and observer.

Time ordering/simultaneity depends on relative velocity between observers, not on their positions.

Length in a reference frame := difference between coordinate positions at the same time.

Different inertial frames (with relative velocity): different coordinate positions, since "same time" different $\Rightarrow$ Lorentz-Fitzgerald contraction:

$$l' = \sqrt{1 - \frac{v^2}{c^2}}\, l = \frac{1}{\gamma} l_0 \ .$$

If no global reference frame: synchronized clocks not available. Operational *length definition*: Observer 1 sends signal to (unintelligent) observer 2 who sends signal immediately back. $L := \frac{1}{2} c \Delta t$.

Assumption: c constant in all reference frames.

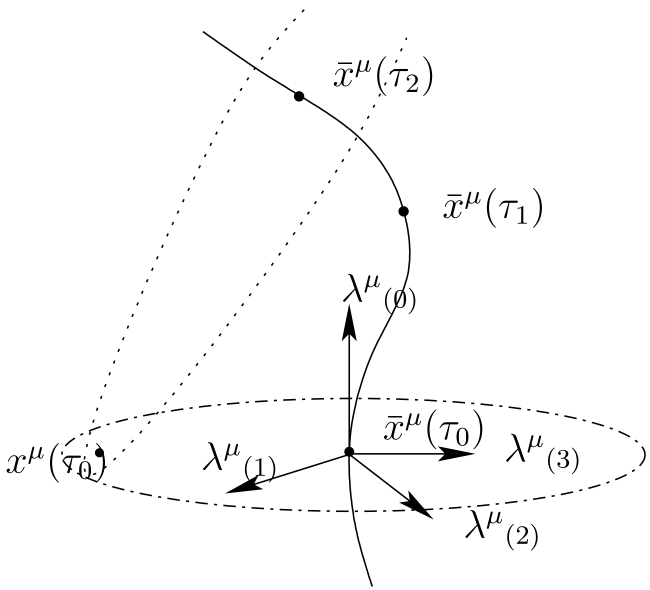
Acceleration lengths

Orthonormal frame field $\lambda^{\mu}{}_{(\alpha)}(\tau)$. Covariant derivative of the frame field:

$$\frac{D\lambda^{\mu}{}_{(\alpha)}}{D\tau} = \Phi_{\alpha}{}^{\beta}(\tau)\lambda^{\mu}{}_{(\beta)} \ .$$

For vanishing non-metricity (using orthonormality): $\Phi_{\alpha\beta}(\tau) = -\Phi_{\beta\alpha}(\tau)$. Therefore:

$$\Phi_{\alpha\beta} := \left[\begin{array}{c|c} 0 & \vec{g}/c \\ \hline -\vec{g}/c & \vec{\Omega} \end{array}\right]$$



New accelerated coordinates using only position and basis frame field:

$$x^\mu(\tau) = \bar{x}^\mu(\tau) + X^i \lambda^\mu{}_{(i)}(\tau)$$

Metric for accelerated observer in Minkowski spacetime:

$$ds^2 = o_{\mu\nu}\, dx^\mu\, dx^\nu = \left[\left(1 + \frac{\vec{g}\cdot\vec{X}}{c^2}\right)^2 - \left(\frac{\vec{\Omega}\times\vec{X}}{c}\right)^2\right](dx^0)^2$$

$$- 2\left(\frac{\vec{\Omega}\times\vec{X}}{c}\right)\cdot d\vec{X} dx^0 - \delta_{ij}\, dX^i\, dX^j\ .$$

Scalar invariants of antisymmetric tensor $\Phi_{\alpha\beta}$:

$$\frac{1}{2c^2}\Phi_{\alpha\beta}\Phi^{\alpha\beta} = -\frac{g^2}{c^4} + \frac{\Omega^2}{c^2} \ ,$$

$$\frac{1}{4c^2}\Phi^*_{\alpha\beta}\Phi^{\alpha\beta} = \frac{\vec{g}}{c^2} \cdot \frac{\vec{\Omega}}{c} \ .$$

Proper acceleration lengths $\mathcal{L}$: $\frac{c^2}{g}$ and $\frac{c}{\Omega}$.

Earth surface:

$$\frac{c^2}{g} = \frac{(3 \cdot 10^8 \, \frac{\mathsf{m}}{\mathsf{s}})^2}{9.8 \frac{\mathsf{m}}{\mathsf{s}^2}} \approx 1 \, \mathsf{ly} = 9.46 \cdot 10^{15} \, \mathsf{m}$$

$$\frac{c}{\Omega} = \frac{3 \cdot 10^8 \, \frac{\mathsf{m}}{\mathsf{s}}}{7.272 \cdot 10^{-5} \, \mathsf{s}^{-1}} = 4.1253 \cdot 10^{12} \, \mathsf{m} \approx 27.5 \, \mathsf{AU}$$

Hypothesis of Locality

> Accelerated observers measure the same physical results as a standard observer that has the same position and velocity at the time of measurement.

Clock hypothesis: Restricted hypothesis of locality for time measurements only.

Hypothesis ingrained in *Newton's theory*, a theory for point particles:

All forces and movements are determined by a second order equation of motion. It determines the state of a particle $(\vec{x}, \vec{v})$ once the initial condition is specified.

- **Waves**: $\omega' = \gamma\left(\omega - \vec{v}\cdot\vec{k}\right)$.

  For accelerated observers: v changes.

  Measurement of frequency only possible, if velocity doesn't change too much over a period of the wave: $T\left|\frac{d\vec{v}}{dt}\right| \ll v$.

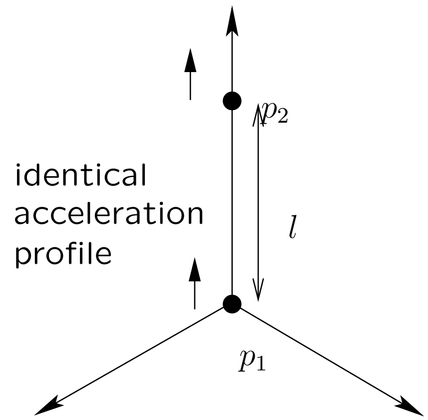
  With $\lambda = cT$, we get $\frac{\lambda}{c}a \ll v < c$, and thus $\lambda \ll \frac{c^2}{a}$.

- **Charged particles**: Accelerated particles radiate. Described by Abraham-Lorentz-Dirac equation

$$m\frac{d^2\vec{x}}{dt^2} - \frac{2}{3}\frac{q^2}{c^3}\frac{d^3\vec{x}}{dt^3} + \cdots = \vec{F}$$

- **Quantum mechanical particles**: have Compton and de Broglie wavelengths associated with them.

Distance measurements: Two observers, a distance l apart, with identical acceleration profiles



- Distance in initial inertial frame: l.
- Distance in momentarily comoving frame: $l' = \frac{1}{\sqrt{1-\frac{v^2(t)}{c^2}}}\, l = \gamma(t) l$
- Distance in accelerating frame with P_1 in origin: $\frac{L}{l'} = 1 - \frac{1}{2}\beta^2\gamma\epsilon + \mathcal{O}(\epsilon^2)$ with $\epsilon = \frac{l}{\frac{c^2}{g}}$.
- Distance in accelerating frame with P_2 in origin: $\frac{L'}{l'} = 1 + \frac{1}{2}\beta^2\gamma_2\epsilon + \mathcal{O}(\epsilon^2)$.

– Distance according to operational definition for P_1:

$$\frac{L^*}{l'} = 1 - \frac{1}{2}\gamma\epsilon(1+\beta^2) + \mathcal{O}(\epsilon^2)$$

.

– Distance according to operational definition for P_2:

$$\begin{aligned}\frac{L'^*}{l'} &= 1 - \frac{1}{2}\gamma\epsilon(1-\beta^2) + \mathcal{O}(\epsilon^2)\\ &= 1 - \frac{1}{2}\frac{\epsilon}{\gamma} + \mathcal{O}(\epsilon^2)\ .\end{aligned}$$

Unruh effect, quantum invariance

Accelerated reference frames are local in nature.

Unruh effect: predicts that accelerated observers see thermal spectrum of particles. The effect is derived by Bogoljubov transformations between nonlocal accelerated and inertial frames.

Circularly polarized electromagnetic wave, frequency ω. Uniformly rotating observer with angular velocity Ω sees (upper sign: RCP):

$$\omega^* = \gamma(\omega \mp \Omega) = \gamma\omega\left(1 \mp \frac{\Omega}{\omega}\right) , \qquad \frac{\Omega}{\omega} = \frac{\lambda/2\pi}{c/\Omega} = \frac{\lambda/2\pi}{\mathcal{L}} .$$

We can choose an angular velocity Ω so that ω^* is zero, i.e. electromagnetic field constant in time. The photon of the inertial frame disappears in the uniformly rotating frame.

Alternatives to Hypothesis of Locality in EM theory

1. **Mashhoon model**: The field that an accelerated observer actually measures depends linearly, but nonlocally on inertial measurements:

$$\mathcal{F}_{\alpha\beta}(\tau) = F_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')\, F_{\gamma\delta}(\tau')\, d\tau' \,,$$

   Kernel K is expected to depend on the acceleration of the observer.

   Investigations: Determining Maxwell's equations for the accelerated observer (they are integro-differential equations).

If K is of convolution type: Volterra calculus can be used.

Concrete example for a uniformly rotating observer:

$$\begin{aligned} \boldsymbol{\mathcal{E}} &= \hat{\boldsymbol{E}} + \int_{\tau_0}^{\tau} \left[\boldsymbol{\omega} \times \hat{\boldsymbol{E}}(\tau') - \frac{\boldsymbol{a}}{c} \times \hat{\boldsymbol{B}}(\tau') \right] d\tau' \,, \\ \boldsymbol{\mathcal{B}} &= \hat{\boldsymbol{B}} + \int_{\tau_0}^{\tau} \left[\frac{\boldsymbol{a}}{c} \times \hat{\boldsymbol{E}}(\tau') + \boldsymbol{\omega} \times \hat{\boldsymbol{B}}(\tau') \right] d\tau' \,, \end{aligned}$$

with $\boldsymbol{a} = (-c\beta\gamma^2\,\Omega,\, 0,\, 0)$ and $\boldsymbol{\omega} = (0,\, 0,\, \gamma^2\,\Omega)$.

2. **Charge & Flux model**: The constitutive relation is linear, but nonlocal:

$$\mathcal{H}^{\alpha\beta}(\tau,\xi) = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu} \int \mathcal{K}_{\mu\nu}{}^{\rho\sigma}(\tau,\tau',\xi) F_{\rho\sigma}(\tau',\xi)\, d\tau' \,,$$

ξ depends on the medium. The kernel will be acceleration-dependent, if we use the connection of the accelerated observer:

$$\mathcal{H}^{\alpha\beta}(\tau) = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu} \Big[F_{\mu\nu}(\tau)$$

$$- c \int_{\tau_0}^{\tau} [\Gamma_{0\mu}{}^{\rho}(\tau-\tau') F_{\rho\nu}(\tau') + \Gamma_{0\nu}{}^{\rho}(\tau-\tau') F_{\mu\rho}(\tau')]\, d\tau' \Big] \,,$$

Concrete example for a uniformly rotating observer:

$$\boldsymbol{D} = \boldsymbol{E} + \int_{\tau_0}^{\tau} \left[\boldsymbol{\omega}(\tau - \tau') \times \boldsymbol{E}(\tau') - \frac{\boldsymbol{a}(\tau - \tau')}{c} \times \boldsymbol{B}(\tau') \right] d\tau' \,,$$

$$\boldsymbol{H} = \boldsymbol{B} + \int_{\tau_0}^{\tau} \left[\boldsymbol{\omega}(\tau - \tau') \times \boldsymbol{B}(\tau') + \frac{\boldsymbol{a}(\tau - \tau')}{c} \times \boldsymbol{E}(\tau') \right] d\tau' \,.$$

For *constant* $\boldsymbol{a}$ and $\boldsymbol{\omega}$, the two models are the same, provided we identify $\mathcal{H}$ with $\mathcal{F}$.

This agreement does *not* extend to the case of nonuniform acceleration.

3. **Electromagnetic potential model**: The actual electromagnetic potential $\mathcal{A}$ relevant for accelerated observers depends linearly, but nonlocally on the inertial potential:

$$\mathcal{A}^\nu = \sqrt{-g}\, g^{\nu\mu} \left[A_\mu + c \int\limits_{\tau_0}^{\tau} \Gamma_{0\mu}{}^{\kappa} A_\kappa \, d\tau' \right] .$$

Concrete example for a uniformly rotating observer:

$$\varphi = \hat{\varphi} - \int\limits_{\tau_0}^{\tau} \frac{\boldsymbol{a}(\tau - \tau')}{c} \cdot \hat{\boldsymbol{A}}(\tau')\, d\tau'$$

$$\boldsymbol{\mathcal{A}} = \hat{\boldsymbol{A}} + \int\limits_{\tau_0}^{\tau} \left[\boldsymbol{\omega}(\tau - \tau') \times \hat{\boldsymbol{A}} - \frac{\boldsymbol{a}(\tau - \tau')}{c}\, \hat{\varphi}(\tau') \right] d\tau' .$$

Equivalence Principle

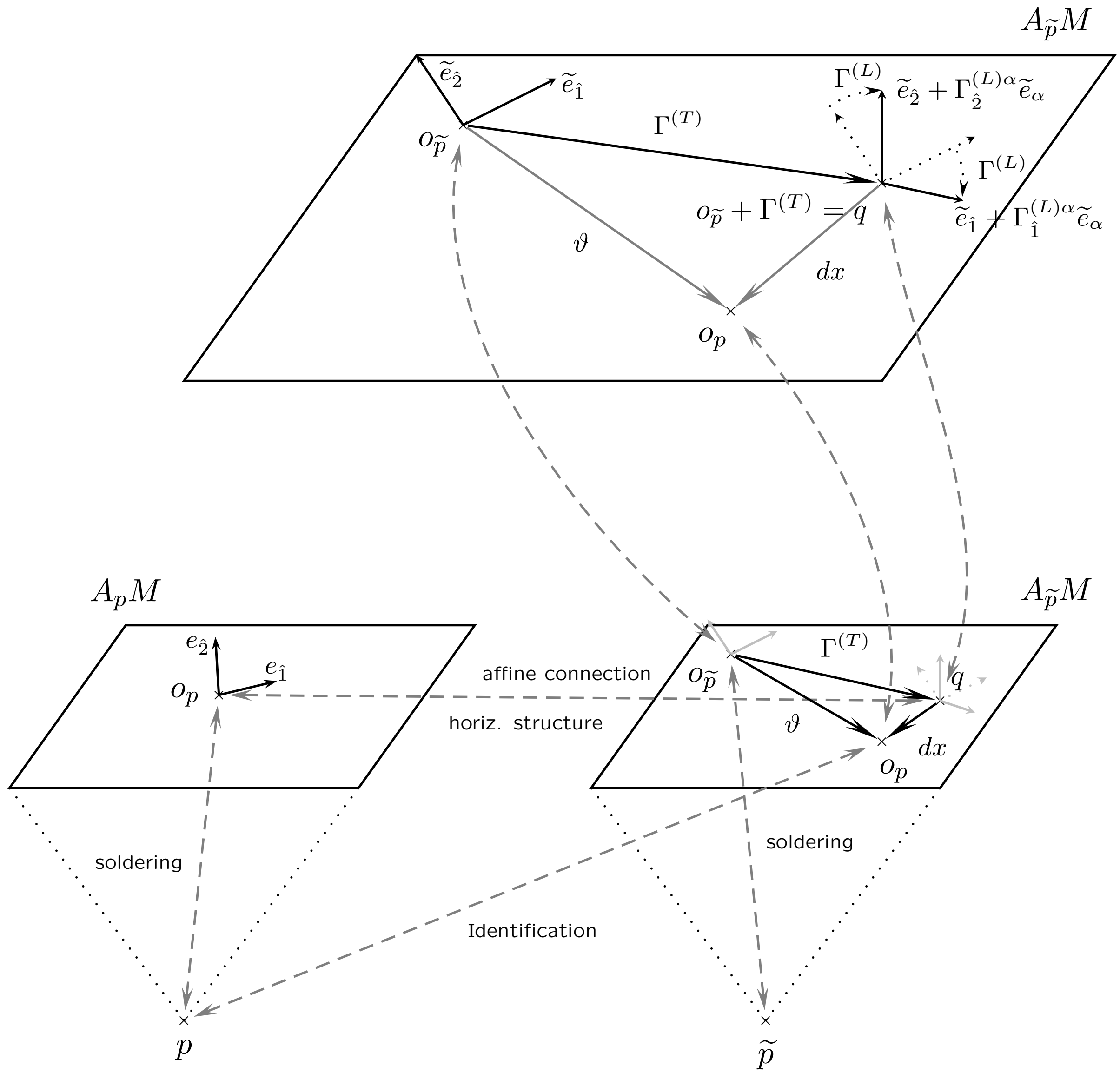
> Observers in a gravitational field and accelerated observers in Minkowski spacetime measure the same physics locally.

The question what accelerated observers measure arises at the core of GR.

Another question: How to connect neighboring affine tangent spaces? Alternative to traditional approach: Removing a different integrability condition by defining a soldering between affine tangent spaces.

Yields a curvature-free manifold with torsion: teleparallel theory.

Soldering and affine connection



Teleparallel theories

Lagrangians *quadratic* in torsion:

$V_{\|} = \frac{1}{2\ell^2}\left(\rho_1\,{}^{(1)}V + \rho_2\,{}^{(2)}V + \rho_4\,{}^{(4)}V\right)$, with:

$${}^{(1)}V = T^\alpha \wedge {}^\star T_\alpha \qquad \text{(pure Yang-Mills type)}\ ,$$

$${}^{(2)}V = \left(T_\alpha \wedge \vartheta^\alpha\right) \wedge {}^\star \left(T_\beta \wedge \vartheta^\beta\right)\ ,$$

$${}^{(4)}V = \left(T_\alpha \wedge \vartheta^\beta\right) \wedge {}^\star \left(T_\beta \wedge \vartheta^\alpha\right)\ .$$

This Lagrangian is *equivalent* to Einstein's theory for

$$\rho_1 = 0, \quad \rho_2 = -\frac{1}{2}, \quad \rho_4 = 1\ .$$

Plan for Ph.D. thesis

- Lengths measurements for rotating observers
- Investigation of Unruh effect
- Determine Maxwell's Equations for Mashhoon model
- Comparison of different nonlocal alternatives
- Relation between PPN parameters and different teleparallel models